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Lab 10 Frequency Response of Filter Circuits

With an Introduction to Automating Instrumentation using Agilent VEE

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1 Purpose

The purpose of this lab is to investigate the frequency response of a lowpass filter (LPF) and a bandpass filter (BPF). In addition, you will be introduced to automating instrumentation using Agilent VEE.

1.1 Equipment

- Agilent 34401A Digital Multimeter
- Agilent 33120A Function Generator
- Agilent 54600B Oscilloscope or Agilent 54622A Deep Memory Oscilloscope
- Agilent E3631A DC Power Supply
- Agilent VEE 6.0 (or 5.0)
- A Capacitance Meter

2 Frequency Domain Representation

Consider an AC circuit with a single voltage source and any number of resistors, capacitors, and inductors. Recall that "AC" is an acronym for *Alternating Current* and the phrase "AC circuit" implies that all voltages and currents in the circuit are sinusoidal. If the frequency of the source is fixed, then a complete analysis in either the time domain or the frequency domain is possible. In the time domain, a differential equation is extracted from the circuit and solved. In general, the order of the differential equation is equal to the number of energy storage elements (L's and C's) in the circuit. A much easier method is to solve the circuit using phasor analysis in the frequency domain. The reason analysis is easier in the frequency domain is because differentiation in time transforms to multiplication by $j\omega$. As a result, an algebraic equation arises rather than a differential equation. Algebraic equations are generally easier to solve than differential equations.

If the frequency of the voltage source is varied, the impedance of each energy storage element changes, and the response of the circuit varies as a function of the input frequency. The frequency response of a circuit is a quantitative description of its behavior in the frequency domain.

2.1 Reasons for Frequency Response Analysis

Frequency response analysis is important to us for two primary reasons. First, if we know the frequency response then we can predict the response of the circuit to any input. Sinusoidal waveforms have the elegant property that they can be combined to form other (non-sinusoidal) waveforms. For example, a 50 Hz triangle wave can be expressed as a sum of sinusoids whose frequencies are integer multiples of 50 Hz (called the Fourier Series representation). Therefore the frequency response allows us to understand a circuit's response to more complex inputs. Second, we are often interested in designing circuits with particular frequency characteristics. For example, in the design of an audio 3-way loudspeaker system, we would like to direct low frequency signals to the woofers, high frequency signals to the tweeters, and mid-frequency signals to the mid-range speakers. Therefore we would need a circuit that is capable of passing certain frequencies of a signal and rejecting others.

2.2 Filters

Filters are frequency selective circuits. The filter used in audio loudspeakers to direct high and low frequency signals to the appropriate transducers is called a cross-over network. There are numerous applications of filters including radio receivers, television receivers, noise reduction systems, and power supply circuits to name just a few.

Circuits that pass certain frequencies and attenuate (eliminate) other frequencies are called filters. Filters are categorized into three general types. Circuits that pass low frequency signals are called low-pass filters (LPF). Circuits that pass signals with frequencies between lower and upper limits (ie. a band of frequencies) are called bandpass filters (BPF). Circuits that pass high frequency signals are call high-pass filters (HPF). Often filters are described in terms of **ideal** characteristics. An ideal lowpass filter, for example, might pass frequencies below 1 kHz and completely stop (reject, eliminate) all frequencies above 1 kHz. A physical LPF filter, however, might pass all frequencies below 1 kHz but also partially pass frequencies above 1 kHz. A physical filter can be designed to closely approximate an ideal filter if needed. However, the better the approximation, the higher the cost because more components are required.

3 Frequency Response

Frequency response is defined as the ratio of the phasor output to the phasor input where the output and input may be either a voltage or a current. The most common ratio is phasor output voltage to phasor input voltage.

$$H(\omega) = \frac{V_{out}}{V_{in}} \tag{1}$$

H(ω) is often referred to as the voltage transfer function. You may refer to H(ω) as either the frequency response or the voltage transfer function.

Figure 1 shows a comparison between the frequency response of a 1st order (contains 1 energy storage element) physical lowpass filter and an ideal lowpass filter.

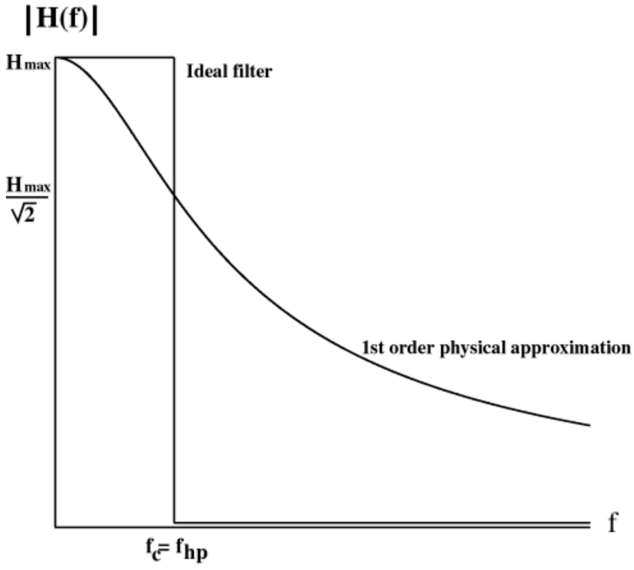


Figure 1: Comparison of the frequency responses of an ideal lowpass filter and a lst order physical approximation.

4 Agilent VEE and the Frequency Response Program

4.1 VEE Software Overview

Agilent's Visual Engineering Environment (VEE) is a graphical programming system that is designed for data acquisition, data analysis, and instrument control. VEE can run on both Microsoft Windows and HP-UX platforms, and is transportable from one system to another.

Programming an application in VEE is very different from programming in a text-based language such as C or BASIC. VEE uses graphical symbols, known as *objects*, to describe programming actions. Data flow is implemented by "wiring" objects together in the desired sequence. Due to VEE's graphical user interface, it is often much easier to get started using it than a typical language. Many engineers and scientists that would not normally try to program an application can get usable output easily with VEE.

VEE programs have a *Detail View* and an optional *Panel View*. The *Detail View* contains the actual program code consisting of objects wired together. Objects can be grouped together into *UserObjects*, or, alternatively, they can be made into subroutines known as *UserFunctions*. Once the program code has been written, a *Panel View* can be added to the program to "hide" the details of the program from the operator and make the program easier to use. The *Panel View* often consists of user-interface objects that mimic actual instrument controls, such as knobs, buttons, displays, and charts, that the user interacts with using either the mouse or the keyboard. The user-interface objects are tied into *Detail View* objects to provide inputs and outputs to the program which will be easily accessible to the user.

4.2 Frequency Response Program

It is fairly easy to determine the frequency response (magnitude) of a circuit in the lab using just a function generator and a multimeter. However, it is much more convenient to automate the measurements and record the measured values using VEE. Having the results stored in the computer memory provides the additional advantage of being able to perform subsequent analysis of the data. In this lab, we will use the Frequency Response program in VEE to determine the frequency response of two filter circuits. The Frequency Response program communicates with the function generator and the multimeter to measure a circuit's frequency response. The program executes the following steps:

1. Configure the multimeter to measure AC voltage.

2. For Loop: for i=0:N-1 Step through frequencies $f_{start} + i f_{step}$

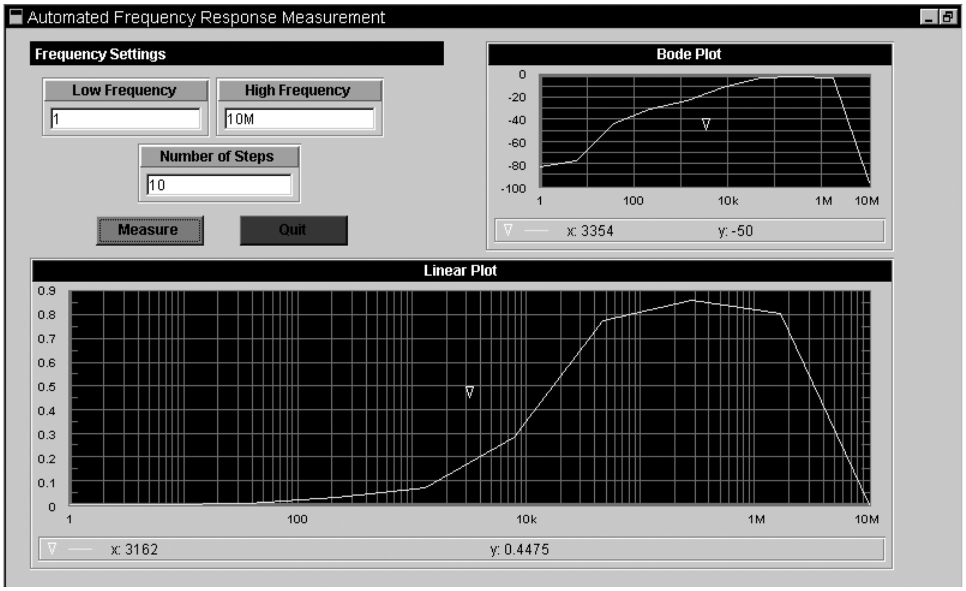
   - Set frequency of function generator.
   - Read the AC Voltage from the multimeter.
   - Form the ratio of the output voltage amplitude to the input voltage amplitude.
   - Output the measured frequency response value to the plots on the front panel.

At each frequency, a measurement of the frequency response is made and the value plotted on the front panel. At the end of the "for loop," N measurements of the frequency response have been made.

4.3 Initial Test of the Frequency Response Program

To open the Frequency Response program do the following:

1. Turn on the monitor and the computer.
2. <ADDITIONAL STEPS SHOULD BE INSERTED HERE DEPENDING ON THE HARDWARE/SOFTWARE CONFIGURATION>\


Automated Frequency Response Measurement

Frequency Settings

Low Frequency: 1

High Frequency: 10M

Number of Steps: 10

Bode Plot: x: 3354 y: -50

Linear Plot: x: 3162 y: 0.4475

Figure 2: Frequency Response Panel in VEE

The screen should now look something like Figure 2. Before connecting the instruments to a circuit, we will perform an initial test of the program.

4.3.1 Initial Test Procedure

1. Turn on the function generator and the multimeter. Configure the multimeter to measure voltage (AC).
2. Connect the multimeter to the function generator.
3. Connect the oscilloscope to the function generator and multimeter.
4. On the Frequency Response program Panel View: set the **High** frequency to 10MHz (type "10M") and the **Low** frequency to 100 Hz. Set the number of steps to 20. Select **Run/Resume** in the Debug menu.
5. Note the displays on the front panel of the function generator and the multimeter. You should see the frequency displayed on the function generator increasing as well as changing values measured by the multimeter. The scope should be showing a frequency-stepped signal. You will have to press the autoscale button on the scope periodically.

Once the program has completed its execution, something similar to a white horizontal line should appear on both plots with an amplitude ≈ 1 on the semi-log plot and ≈ 0 dB on the log-log (Bode) plot.

Both the semi-log and log-log plots include graph cursors to aid determination of particular frequencies. Each graph has a cursor which can be moved by clicking on it with the mouse and dragging it across the measured plot data. The cursor's X/Y position is indicated just below the graph. Move the cursor so that the X location is around 1000 Hz.

5 Frequency Response of an RC Lowpass Filter

In this section we consider the frequency response of an RC lowpass filter. The filter circuit, shown in Figure 3, consists of a resistor and a capacitor in series with the function generator. The function generator has an internal resistance of 50 Ω which you must take into account in the analysis. **Recall that the voltage v_s is two times the voltage displayed on the front panel of the function generator**. The output of the circuit is taken to be the voltage across the capacitor.

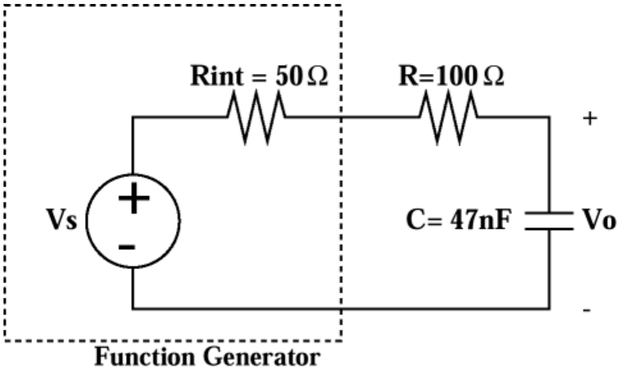


Figure 3: RC Lowpass Filter Circuit

The behavior of the circuit as a function of frequency may be deduced from considering the impedance of the capacitor for different frequencies. For example, at DC (f=0) the capacitor is an open circuit. Therefore, no current flows through any of the elements since they are all in series with the capacitor. Since the current through the resistance is zero, the voltage across them is zero. KVL applied around the loop shows $v_o = v_s$. If the frequency is arbitrarily large ($f \to \infty$) the capacitor becomes a short circuit and therefore the output voltage $v_o = 0$. As the frequency increases from DC (f=0) the capacitor goes from being an open circuit to being a short circuit. As a consequence, the output voltage goes from v_s to zero. The circuit produces its greatest response at DC. As the frequency is increased, the response drops. As the frequency is increased further the response drops to zero. Low frequencies pass, high frequencies are cut. It is a low-pass filter (LPF).

5.1 Derivation of the Frequency Response

Combining the impedances $(Z_R = R,\ Z_C = 1/j\omega C)$ of the elements gives a total impedance of $R_{int} + R + Z_c$. The output voltage across the capacitor is given by Equation 2

$$V_O = \frac{V_S Z_C}{R_{\text{int}} + R + Z_C} = \frac{V_S(\frac{1}{j\omega C})}{R_{\text{int}} + R + \frac{1}{j\omega C}} \tag{2}$$

Multiplying both numerator and denominator by $j\omega C$ for simplification gives:

$$V_o = \frac{V_s}{1 + jwC(R_{int} + R)} \tag{3}$$

Dividing Equation 3 by V_s gives Equation 4, the voltage transfer function (or frequency response) of the circuit expressed in terms of angular frequency.

$$H(w) = \frac{V_o}{V_s} = \frac{1}{1 + j\omega C(R_{\text{int}} + R)} \tag{4}$$

Replacing ω with $2\pi f$ gives Equation 5, the transfer function expressed in terms of frequency.

$$H(f) = \frac{1}{1 + j2\pi f C(R_{\text{int}} + R)} \tag{5}$$

The magnitude of the voltage transfer function, called the gain is

$$|H(f)| = \frac{1}{\sqrt{1 + \left[2\pi f C\left(R_{\text{int}} + R\right)\right]^2}} \tag{6}$$

The limit of |H(*f*)| as f approaches infinity is zero as expected. As $f \to 0$ the magnitude approaches 1. A graph of this function looks like the 1st order approximation filter response in Figure 1. In the case of the ideal low-pass filter, it is clear where the passband ends and the stopband begins. It is not clear at all where the passband ends and the stopband begins in the case of the 1st order RC LPF. It is by convention that we define the band edge using the “half-power” frequency f_{HP}. When the output power drops to one-half of its maximum value, the output response (whether voltage or current) drops to $\frac{1}{\sqrt{2}} H_{max}$ where H_{max} is the maximum gain. To find the half-power frequency, you take the expression for the gain (equation 6) and set it equal to $\frac{1}{\sqrt{2}} H_{max}$ and solve for f_{HP}. This is the frequency at which $|H(f)|^2 = \frac{1}{2}$ or $|H(f)| = \frac{1}{\sqrt{2}}$ since H_{max} =1 for this circuit. This is also called the cutoff frequency f_c of the lowpass filter and is given by Equation 7.

$$f_{HP} = f_c = \frac{1}{2\pi C\left(R_{int} + R\right)} \tag{7}$$

5.2 Measurement of the Frequency Response

You will now measure the frequency response of the lowpass filter in Figure 3 experimentally. First, you will make a series of measurements **manually**. After the **manual** measurements have been taken, you will then make similar measurements automatically using VEE.

1. Obtain a 100 Ω resistor and a 47 nF **mylar** capacitor from the parts bin. Measure (and record) the exact capacitance with the capacitance meter in the lab, as well as the resistance using the ohmmeter. **After using the capacitance meter, always turn off its power or the battery will die.**
2. Connect the function generator to the series combination of the resistor and capacitor. Reset the function generator (Turn it off. Wait a few seconds and then turn it on again.) so that it can be set manually. Set the amplitude to 1.0 volt peak-to-peak (on the front display panel). Set the frequency to the values given in Table 1 and record the AC voltage measured across the capacitor using the multimeter set to measure AC voltage.

3. The maximum response should occur at DC (f=0 Hz) but cannot be measured with the function generator as the source because it is incapable of generating 0 Hz. However, the value of the response you measure at f = 100 Hz should be nearly identical to the DC response. Using the value of the response you measured at 100 Hz, adjust the frequency of the function generator until the response drops to $\frac{1}{\sqrt{2}}$ times the response at 100 Hz. Record that frequency as the measured cutoff (half-power) frequency f_c.

You will now measure the frequency response of the lowpass filter circuit **automatically** using the Frequency Response program in VEE.

Frequency	AC Voltage V_o
100 Hz	
1 kHz	
10 kHz	
20 kHz	
25 kHz	
30 kHz	
35 kHz	
40 kHz	
50 kHz	

Table 1: Lowpass filter experimental frequency response

1. Set the **High** frequency to 50 kHz and Low frequency to 100 Hz.
2. Set the **Number of Steps** to 50.
3. Select **Run/Resume** in the **Debug** menu.
4. Observe the function generator (and oscilloscope trace) stepping through different frequencies from 100 Hz to 50 kHz and the multimeter measuring the corresponding responses.
5. When the simulation is complete, record the value of the DC response which is taken to be the value of the response at 100 Hz. In your lab report, justify this assumption. You will do this by comparing the value of the theoretical frequency response magnitude at DC (0 Hz) and at f=100 Hz. Divide this value by $\sqrt{2}$ to determine the response at the half-power frequency.
6. Use the graph cursors to determine f_c.
7. Run the program again by pressing the **Measure** button, this time narrowing the frequency range to focus on the frequencies near the cutoff frequency. Observe the traces on the scope as the frequency is stepped. Use the graph cursors to determine f_c again.

Comment on the accuracy of the two measurements of f_c made using VEE, as well as the value obtained **manually**. Hint: Consider the nature of the three measurements. When you used VEE to automate the measurements, 50 frequencies were applied and the response at those frequencies were obtained. When you narrowed the range of frequencies, the measurements were **closer** together in the frequency domain. However, there were still only a finite number of frequencies utilized. When you measured the half-power frequency f_c

manually, you varied the frequency by hand until the response dropped to $\frac{1}{\sqrt{2}}$ times the DC response.

6 Frequency Response of a Bandpass Filter (BPF)

We now turn our attention to the frequency response of the parallel resonant bandpass filter (BPF) circuit shown in Figure 4. Recall that a bandpass filter passes frequencies between two limits. The range of frequencies between the two limits is refered to as the passband of the filter and the difference between the limiting frequencies as the **bandwidth** Δf of the filter.

The magnitude of the transfer function of this circuit is given by Equation 8 and the resonance frequency f_r by Equation 9. Resonance is a physical phenomenon in which stored energy oscillates between two energy storage elements. In the BPF circuit shown in Figure 4, the magnetic energy stored in the inductor and the electric energy stored in the capacitor are exchanged back and forth in harmony at resonance. Resonance occurs at a single frequency f_r determined by the values of L and C. The frequency response magnitude

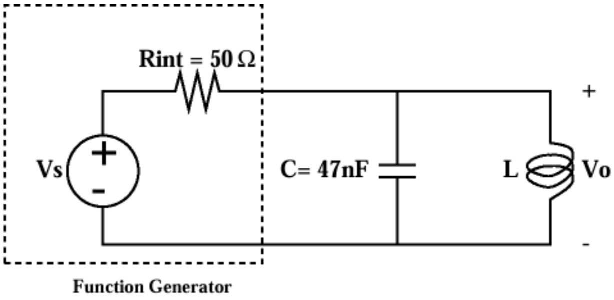


Figure 4: Parallel Resonant Bandpass Filter Circuit

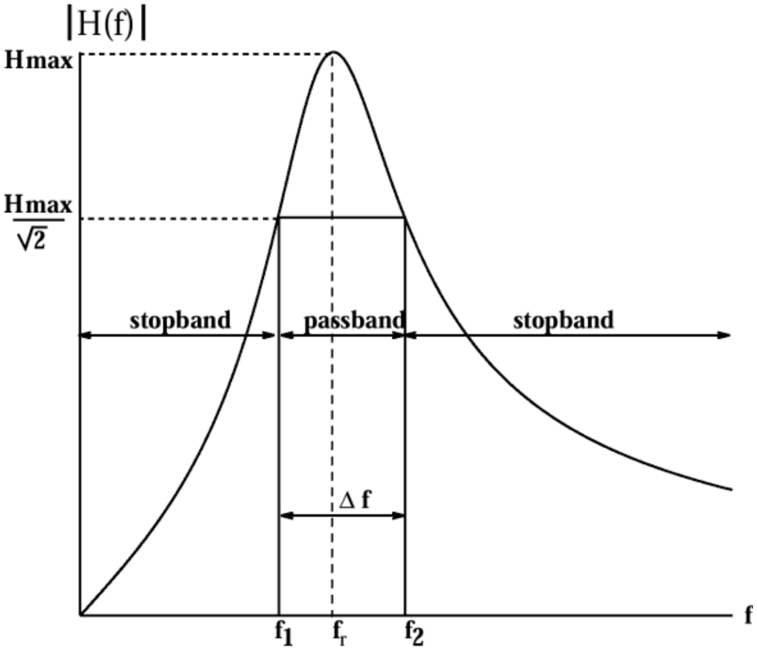


Figure 5: Frequency Response for a Parallel Resonant Bandpass Filter Circuit

attains a maximum at the resonant frequency. A graph of the frequency response magnitude for the parallel resonant BPF is shown in Figure 5.

$$|\mathrm{H}(\mathrm{f})| = \left|\frac{V_o}{V_s}\right| = \frac{1}{\sqrt{1+\left[R_{int}(2\pi f C - \frac{1}{2\pi f L})\right]^2}} \qquad (8)$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \qquad (9)$$

1. Determine f_r using C = 47 nF and the estimated value of L for your inductor found in Lab 7.
2. You will now manually determine the frequency response of the bandpass circuit in Figure 4. Connect the function generator across the parallel combination of the 47 nF capacitor and the inductor you made in Lab 5 (note that you will have to do this twice, once for each student's inductor). Reset the function generator by cycling the power. Set the amplitude (displayed) to 1.0 V_{pp}. Set the frequency to the values shown in Table 2 and record the AC voltage measured across the parallel combination of inductor and capacitor.
3. Adjust the function generator until the measured voltage is a maximum. Record the resonance frequency, f_r. Determine the inductance of your coil using this frequency.
4. Determine the frequency response of the bandpass circuit in Figure 4 using VEE.
    - Set the **High** frequency to 1 MHz and Low to 1 kHz. Set the number of steps to 50. Select **Run/Resume** in the **Debug** menu and obtain an initial response plot.
    - Change to frequency limits to zoom in on the resonance region.
    - Obtain values for the resonance frequency f_r, and the half-power frequencies f_1 and f_2. Determine the bandwidth $\Delta f = f_2 - f_1$ and the Q of the circuit given by Equation 10. Determine the inductance of your coil using the resonance frequency found with VEE.

Frequency	AC Voltage V_o
1 kHz	
10 kHz	
50 kHz	
100 Hz	
200 kHz	
300 kHz	
400 kHz	
500 kHz	
1 MHz	

Table *2:* Bandpass filter experimental frequency response

7 Pspice (to be done outside of lab)

7.1 Lowpass Filter Circuit

Simulate the lowpass filter circuit shown in Figure 3 using PSpice. Be sure to include the internal resistance of the function generator. Examine the circuit EX3 in the PSpice tutorial in the EE61 Course-Pak to see how to produce plots of frequency response. Compare your simulation results to theory and to your measured frequency response.

7.2 Bandpass Filter Circuit

Simulate the bandpass filter circuit shown in Figure 4 using PSpice. Be sure to include the internal resistance of the function generator. Obtain a plot of the frequency response magnitude for the bandpass filter circuit, and compare your simulation results to theory and to your measured frequency response.

8 Questions

1. Derive Equation 8 using impedances ($Z_R = R$, $Z_C = 1/j\omega C$ and $Z_L = j\omega L$). From Equation 8 show that the resonance frequency f_r is given by Equation 9.
2. Determine which of the three values of inductance for your coil is most accurate 1) the value estimated in Lab 5 using the time constant of an RL switched circuit, 2) the value estimated using the resonant frequency you obtained using manual frequency response measurements, or 3) the value estimated using the resonant frequency you obtained using VEE to automatically measure the frequency response. Justify your answer and explain why the other values are less accurate.